Published by Institute of Physics Publishing for SISSA
Received: *January 8, 2007*
Accepted: *February 13, 2008*
Published: *February 26, 2008*

Holography of non-relativistic string on $\mathrm{AdS}_5 \times \mathrm{S}^5$

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Abstract: We discuss a holographic dual of a non-relativistic (NR) string on $\mathrm{AdS}_5{\times}\mathrm{S}^5$. The NR string can be regarded as a semiclassical string around an AdS_2 classical solution corresponding to a straight Wilson line in the gauge-theory side. The quadratic action with respect to the fluctuations is composed of free massive and massless scalars, and free massive fermions on the AdS_2 world-sheet. We show that the complete agreement of the spectra between the NR string and a conformal quantum mechanics (CQM). Then we show a holographic relation between normalizable modes of the NR string and wave functions in the CQM. Then it may be argued from this result that an $\mathrm{AdS}_2/\mathrm{CFT}_1$ would be realized in $\mathrm{AdS}_5/\mathrm{CFT}_4$. We can really discuss a GKPW-type relation by considering non-normalizable modes of the NR string in Euclidean signature. Those modes give a source term insertion to the Wilson line, which can also be regarded as a small deformation of it.

Keywords: AdS-CFT Correspondence, Conformal Field Models in String Theory, Long strings.

JHEP02(2008)092

Contents

1. Introduction

One of the most important subjects in string theory is AdS/CFT correspondence [1, 2], which gives a realization of holography within the framework of string theory. There is no rigorous proof of it. An important issue toward the proof is to quantize type IIB string on $AdS_5 \times S^5$. Although it is shown to be classically integrable [3], its action constructed in [4] is so non-linear that it is difficult to quantize it directly. Thus it may be important to look for a solvable limit. In fact, the study of pp-wave string [5, 6] with Penrose limit [7] played an important role in examining AdS/CFT even in a non-BPS region [8].

Recently an interesting solvable subsector of $\mathrm{AdS}_5/\mathrm{CFT}_4$ with non-relativistic (NR) limit[1] was proposed in [11]. After taking the limit, the full string theory is reduced to a free theory on the AdS_2 world-sheet. More precisely speaking, the resulting theory contains three massive and five massless scalars, and eight massive fermions. It can also be obtained as a semiclassical limit of the full AdS string around an AdS_2 solution corresponding to a Wilson line [12], like in the case of pp-wave string [13]. The quadratic action of string and D-branes in the semiclassical limit are computed in [14 − 17].

In this paper we proceed with the previous works [15 − 17] and discuss a holographic dual of the NR string. Here we are confined to the case of straight Wilson line. Then the physical spectrum of the NR string has already been computed in [18] and it is used to evaluate the semiclassical partition function of AdS superstring in [14]. Motivated by this spectrum and the AdS_2 world-sheet geometry, it would be worth looking for a candidate of the dual description of the NR string. The dual theory may be defined on the Wilson line which is the boundary of the world-sheet and one-dimensional. The physical normalizable modes of the NR string decay before reaching the boundary of the AdS_2 world-sheet. However, it may be related to the quantum Hilbert space according to the Lorentzian AdS/CFT dictionary [19].[2]

In fact we can find that the spectrum of a one-dimensional conformal quantum mechanics (CQM) [21][3] completely agrees with that of the NR string under a certain identification of parameters. The wave functions of the CQM can be reproduced from the physical modes of the NR string following [23]. In addition the two-point function in the CQM can also be reproduced by evaluating the Wightman function from the bulk NR string.

Based on the evidence we may argue an $\mathrm{AdS}_2/\mathrm{CFT}_1$ realized in $\mathrm{AdS}_5/\mathrm{CFT}_4$ via the so-called "double holography." Eventually, it would not be so surprising to find the holography of this type, since the holographic relation of this type has already been observed in the case of defect CFT [24]. It would be helpful for the readers familiar with the case to imagine replacing $\mathrm{AdS}_4\times\mathrm{S}^2$ with simply AdS_2 .

Then we can discuss a GKPW-type relation [2] between the NR string and the Wilson loop with a source term insertion by considering non-normalizable (NN) modes of the NR string. This source term is given by a one-dimensional integral and sensitive to the NN modes on the AdS_2 string world-sheet. The operators coupled to the boundary values of the NN modes have already been clarified in [16, 17]. The insertion of the source term can be regarded as a deformation of the Wilson line. Furthermore we discuss supersymmetries preserved under the insertion of the source term.

This paper is organized as follows. In section 2 we first introduce the action of the NR string on $\mathrm{AdS}_5\times\mathrm{S}^5$. Then we discuss the physical spectrum of the NR string. This is a review of the work [18]. In section 3 we find a CQM dual to the NR string. This section

[1] A non-relativistic limit for a closed string in flat space was discussed in [9, 10] earlier than in $\mathrm{AdS}_5\times\mathrm{S}^5$.

[2] For a more comprehensive discussion on the Lorentzian AdS/CFT see [20].

[3] For a recent review of CQM see [22].

contains some reviews on the basics of CQM. In section 4 we first remember a semiclassical derivation of the bosonic part. Then we discuss the treatment for the divergent part, the relation between the two methods: 1) adding a constant B-field and 2) the standard method to consider a Legendre transformation. Next we discuss a holographic relation, i.e., GKPW-type relation between the NR string and a straight Wilson line with a source term insertion by considering NN modes of the NR string. We carefully examine the relation between the fluctuations and the NN modes, and we show that the fluctuations do not diverge at the boundary even for the NN modes. Section 5 is devoted to a conclusion and discussions.

2. NR string on $\mathrm{AdS}_5 \times \mathrm{S}^5$

We introduce a non-relativistic (NR) string on $\mathrm{AdS}_5{\times}\mathrm{S}^5$ [11]. In the first place we shall give a brief review of a series of papers about the action of the NR string on $\mathrm{AdS}_5{\times}\mathrm{S}^5$. Our convention and notation are also described. Then the physical spectrum of the NR string obtained in [18] is briefly reviewed here.

2.1 The action of NR string

The NR limit concerned here is realized by taking the speed of light transverse to an AdS_2 subspace to be infinite,[4] while it should be kept along the AdS_2. The concrete prescription to take the limit is given in appendix A.

After taking the NR limit and fixing the κ-symmetry, the action is reduced to the following form,[5]

$$S^{(\mathrm{NR})} = -\int d^2\sigma\sqrt{-g}\left[\frac{g^{ij}}{2}\partial_i x^a\partial_j x^a + \frac{1}{R_0^2}x^a x^a + \frac{g^{ij}}{2}\partial_i y^{a'}\partial_j y^{a'} - 2i\bar{\theta}_+\Gamma^\mu \mathbf{v}_\mu^i D_i\theta_+\right]. \quad (2.1)$$

Here the string tension has been absorbed by rescaling the variables $(x^a, y^{a'}, \theta_+)$. The world-sheet coordinates are $\sigma^i = (\sigma^0,\sigma^1) = (\tau,\sigma)$ and the world-sheet metric g_{ij} on the world-sheet is

$$g_{ij} = \eta_{\mu\nu}\mathbf{v}_i^\mu\mathbf{v}_j^\nu \quad (\mu,\nu = 0,1)\,, \qquad g \equiv \det g_{ij}\,,$$

where $\mathbf{v}^\mu$ is a zweibein of AdS_2. That is, the world-sheet geometry is a two-dimensional AdS space. The eight bosonic variables x^a $(a = 1,2,3)$ and $y^{a'}$ $(a' = 1,\ldots,5)$ come from AdS_5 transverse to the AdS_2 and S^5 respectively.

The fermionic variable θ is decomposed into the two parts like $\theta = \theta_+ + \theta_-$ in terms of the two eigenvalues of $\Gamma_* \equiv \Gamma_0\Gamma_1\tau_3$ satisfying $\Gamma_*^2 = 1$. That is, $\Gamma_*\theta_\pm = \pm\theta_\pm$. The κ-symmetry is fixed by taking the condition $\theta_- = 0$. Note that the action (2.1) preserves 16 linear supersymmetries and 16 non-linear supersymmetries. That is, $\mathcal{N} = 8$ supersymmetries in two dimensions are preserved [14, 11]. For an earlier discussion on the structure of $\mathcal{N} = 1$ supermultiplet on AdS_2, see [18].

[4] More generally, another NR limit may be considered by making the speed of light in the directions transverse to an $\mathrm{AdS}_p{\times}\mathrm{S}^q$ subspace infinite. In fact, considering 1/2 BPS AdS-branes, NR limits of DBI actions on $\mathrm{AdS}_5{\times}\mathrm{S}^5$ can be discussed [15].

[5] For a heuristic derivation see appendix B.

The Virasoro condition has already been solved and the action apparently contains the eight physical components, i.e., three massive and five massless bosons, and eight massive fermions. The masses of bosons and fermions are measured by the AdS radius R_0 , and the boson mass is $m_{\rm B}^2 = 2/R_0^2$ and the fermion mass is $m_{\rm F}^2 = 1/R_0^2$. Thus the SO(3)×SO(5) bosonic symmetry is preserved rather than SO(8) . The world-sheet has an SL(2) symmetry. Including the fermions the symmetry is enhanced to OSp(4^*|4).

In general, unphysical components may be contained implicitly through the AdS$_2$ zweibein $\mathbf{v}^\mu$. Those can however be removed by taking a static gauge [11] and the zweibein does not depend on the unphysical components. Hereafter we will assume that the static gauge is taken. According to the gauge-fixing to the static gauge, in working on Euclidean AdS$_5$×S^5, the world-sheet AdS$_2$ should also be Euclidean AdS$_2$ (EAdS$_2$) .

2.2 Physical spectrum of NR string

We will discuss the quantization of normalizable solutions of a free scalar field ϕ on AdS$_2$ in Lorentzian signature. It should be regarded as one of the scalar components contained in the action of the NR string. Namely, $\phi = x^a$ or $y^{a'}$. Here we restrict ourselves to the bosonic part, but the argument for the fermion is also given in [18].

The world-sheet AdS$_2$ geometry is described by the metric:

$$ds^2 = g_{ij}dx^i dx^j = \frac{R_0^2}{\cos^2\rho}(-dt^2 + d\rho^2)\,. \tag{2.2}$$

Here the time t is the global time and $-\pi/2 \le \rho \le \pi/2$. The boundary has the topology $\mathbb{R}\times S^0$ and so there are two time-like boundaries at $\rho = \pm\pi/2$.

The classical equation of motion is given by

$$\cos^2\rho\left(-\partial_t^2 + \partial_\rho^2\right)\phi - m^2\phi = 0\,, \tag{2.3}$$

where $m^2 = R_0^2 m_B^2 = 2$ for x^a and $m^2 = 0$ for $y^{a'}$. The formal solution is typically written by the Gegenbauer polynomial[6] $C_\alpha^\lambda(z)$ [18, 23]:

$$\phi_\omega^{\lambda,\pm}(t,\rho) = \mathrm{e}^{\pm i\omega t}(\cos\rho)^\lambda C_{\omega-\lambda}^\lambda(\sin\rho)\,. \tag{2.4}$$

In order for (2.4) to be the solution of (2.3) , ω has to take the discrete value as

$$\omega = n + \lambda \qquad (n = 0, 1, \cdots)\,,$$

and furthermore λ takes Δ or $1-\Delta$, where Δ is defined as

$$\Delta \equiv \frac{1}{2}\left(1 + \sqrt{1+4m^2}\right)\,.$$

Then in order to discuss the normalizability we have to introduce the Klein-Gordon norm

$$(\phi_1, \phi_2) \equiv i\int d\rho\,\sqrt{-g}g^{tt}\left[\phi_1\partial_t\bar{\phi}_2 - \bar{\phi}_2^*\partial_t\phi_1\right]\,. \tag{2.5}$$

[6] For some properties of Gegenbauer polynomials, see appendix C.

The solution (2.4) is normalizable with (2.5) if and only if λ is real and $\lambda > -1/2$. In particular, the reality of λ is equivalent to the Breitenlohner-Freedman (BF) bound [25]:

$$m^2 \geq -\frac{1}{4}. \tag{2.6}$$

Furthermore by imposing the unitarity λ can be further restricted and the additional condition is $\lambda > 0$. Thus we will take λ as Δ.

The quantization can be performed by following the standard procedure. First of all, the field $\phi(t,\rho)$ is expanded as

$$\phi^\Delta(t,\rho) = \sum_{n=0}^{\infty} a_n \phi_n^\Delta(t,\rho) + \sum_{n=0}^{\infty} a_n^\dagger \bar{\phi}_n^\Delta(t,\rho)\,, \tag{2.7}$$

where the n-th mode is defined as

$$\phi_n^\Delta(t,\rho) \equiv c(\Delta)\sqrt{\frac{n!}{\Gamma(n+2\Delta)}}\,\mathrm{e}^{-i(n+\Delta)(t+\pi/2)}(\cos\rho)^\Delta C_n^\Delta(\sin\rho)\,. \tag{2.8}$$

The normalization constant $c(\Delta)$ is defined as

$$c(\Delta) \equiv \frac{\Gamma(\Delta)2^{\Delta-1}}{\sqrt{\pi}}\,,$$

and it has been fixed by the following conditions:

$$(\phi_m^\Delta, \phi_n^\Delta) = \delta_{m,n}\,, \quad (\bar{\phi}_m^\Delta, \bar{\phi}_n^\Delta) = -\delta_{m,n}\,, \quad (\phi_m^\Delta, \bar{\phi}_n^\Delta) = 0\,.$$

Note that the normalizable modes are decaying as approaching the boundary, as easily shown from the behavior of the Gegenbauer polynomials at the boundary:

$$C_n^\Delta(1) = \frac{\Gamma(n+2\Delta)}{\Gamma(2\Delta)n!}\,.$$

We can canonically quantize $\phi(t,\rho)$ and the creation and annihilation operators $a_n^\dagger$ and a_m satisfy the commutation relations:

$$[a_m, a_n^\dagger] = \delta_{m,n}\,, \quad [a_m, a_n] = [a_m^\dagger, a_n^\dagger] = 0\,.$$

Then the Fock vacuum is defined as

$$a_n|0\rangle = 0 \qquad (\forall n = 0, 1, \cdots)\,,$$

and the Fock space $\mathcal{F}^\Delta$ is spanned as

$$\mathcal{F}^\Delta = \bigoplus_{n_1,\ldots,n_k} \mathbb{C}\, a_{n_1}^\dagger \cdots a_{n_k}^\dagger |0\rangle\,.$$

The normalizable modes on AdS_2 in the coordinates (2.2) have been studied well in [18] with the help of supersymmetries. By using them an effective potential of a free scalar field

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on AdS_2 has been computed in [26]. The technique to compute the effective potential has been applied to computing the one-loop vacuum energy of the semiclassical action of AdS superstring in [14]. Note that the Poincare disk should be considered rather than the strip of (2.2) if the circular case is considered.

Here the time coordinate t in the metric (2.2) is the global time and the time translation invariance is associated with the global AdS energy E. The energy E is given as the eigenvalue of the Hamiltonian represented by

$$H = \sum_{n=0}^{\infty} (n+\Delta) a_n^{\dagger} a_n \,, \tag{2.9}$$

up to the zero point energy.[7] Then the energy of the n-th mode is given by

$$E_n = n + \Delta \,. \tag{2.10}$$

In the next section we will find a dual CQM whose spectrum completely agrees with (2.10).

3. The dual of the NR string

We discuss a CQM, which is expected to be dual to the quadratic fluctuations around the AdS_2. Most of the results here have already been given in [21] (For a recent review see [22]). We will give an interpretation of the results in the context of AdS/CFT duality. In particular, the wave functions of the CQM coincide with the quantum states of the scalar fields on AdS_2 [18].

3.1 CQM and its algebra

The Hamiltonian of CQM is given by [21]

$$H = \frac{1}{2}p^2 + \frac{g}{2x^2} \,, \tag{3.1}$$

where g is a coupling constant. This model is often called DFF model.

The one-dimensional conformal group is SO(2,1)=SL($2,\mathbb{R}$) and its Lie algebra contains three generators, H: Hamiltonian, D: dilatation, and K: special conformal. The generators D and K are given by, respectively,

$$D = -\frac{1}{4}\left(px + xp\right) \,, \qquad K = \frac{1}{2}x^2 \,.$$

These generators obey the following relations:

$$[H, D] = iH \,, \quad [K, D] = -iK \,, \quad [H, K] = 2iD \,.$$

[7] The zero-point energy is not important because it must be canceled out together with the contributions from the other degrees of freedom on the AdS_2 due to the supersymmetries.

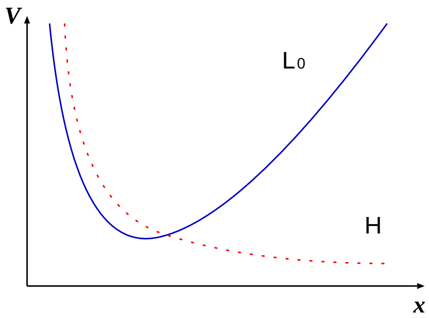

Figure 1: The potentials for H and L_0.

It is awkward to compute the energy spectrum of H because the potential has no minimum and the spectrum is continuous. Hence according to [21] let us change the basis of the algebra from (H, D, K) to (R, D, S), where R and S are defined as, respectively,

$$R \equiv \frac{1}{2}\left(aH + \frac{1}{a}K\right), \qquad S \equiv \frac{1}{2}\left(-aH + \frac{1}{a}K\right). \tag{3.2}$$

Here a is a constant parameter with dimension of length-squared and it can be understood as a mass parameter of the theory described by $1/\sqrt{a}$. When we regard R as another Hamiltonian, the potential has a minimum and its spectrum becomes discrete. For the shapes of the potentials see figure 1.

Then the new algebra is given by

$$[D, R] = iS\,, \quad [S, R] = -iD\,, \quad [S, D] = -iR\,.$$

This is nothing but the SO(2,1) algebra and the generator R corresponds to a compact rotation U(1) $\subset$ SO(2,1). The others describe hyperbolic non-compact rotations. In the context of AdS/CFT, the one-dimensional conformal group SO(2,1) is a subset of the four-dimensional conformal group SO(2,4). Then the compact rotation R corresponds to the time translation symmetry with respect to the global AdS time, which is generated by $H_{\rm full} = \frac{1}{2}(P_0 + K_0)$ (P_0 an K_0 are zeroth components of translation and special conformal generators of the SO(2,4)). This fact plays an important role when considering the correspondence between the spectra of the CQM and a scalar field on AdS$_2$.

3.2 Discrete eigen-value problem

Next let us consider the energy eigen-value problem:

$$R\beta_n = E_n\beta_n \qquad (n = 0, 1, \ldots,)\,.$$

The energy for the normalizable ground state, E_0, is given by

$$E_0 = \frac{1}{2} + \frac{1}{2}\sqrt{g + \frac{1}{4}}\,,$$

and the energy eigen-value E_n is

$$E_n = E_0 + n\,. \tag{3.3}$$

Note that (3.3) completely agrees with the quantum spectrum (2.10) of a scalar field on AdS_2 [18], if we identify the scalar mass with the coupling constant g as follows:

$$g = 4m^2 + \frac{3}{4}\,. \tag{3.4}$$

Then E_0 is nothing but Δ, i.e.,

$$E_0 = \Delta\,.$$

The parameter a appeared in (3.2) should be identified with the square of the AdS radius R_0^2. The energy eigen-states $\{\beta_n\}_{n=0,\cdots}$ correspond to those of the scalar field on AdS_2, namely normalizable modes. In fact, the set of β_n can be reconstructed from quantized scalar fields on the bulk AdS_2, by following [23]. This recipe will be available in the next subsection. Thus the result realizes the claim of [19].

According to (3.4), the massive (massless) scalars with $m^2 = 2$ ($m^2 = 0$) correspond to the CQMs with the following coupling constant:

$$g_{\mathrm{AdS}} = \frac{35}{4} \quad (m^2 = 2,\ \Delta = 2)\,, \qquad g_{\mathrm{S}} = \frac{3}{4} \quad (m^2 = 0,\ \Delta = 1)\,. \tag{3.5}$$

Here we should note the relation between the coordinate system and the basis of the conformal algebra. The Hamiltonian (3.1) is associated with the time translation symmetry in the Poincare time, and R is with the Cartan generator of SO(2,1), which describes the compact rotation. Thus, in the context of AdS/CFT duality, the Hamiltonian (3.1) corresponds to the Poincare energy and the R to the global AdS energy.

An interesting question is whether the CQM with the coupling (3.5) can be derived directly from $\mathcal{N}$=4 SYM. This is an open problem to be investigated in the future, and now we have no answer to this question.

Finally we shall give some comments on the above CQM argument below.

Comment on the bound for the coupling. It has been known that the following bound

$$g \geq -1/4$$

should be satisfied so that the energy spectrum is bounded from below. This statement is obtained from the study of the spectrum of the system (3.1). From the viewpoint of AdS/CFT duality we can easily understand this result as the BF bound for the AdS_2 case (2.6). This is surely consistent with the identification (3.4).

Conformal symmetry. By redefining the dilatation operator[8] as $\tilde{D} = -2D$ and introducing the following linear combinations

$$L_0 = \frac{1}{2}\left(aH + \frac{K}{a}\right)\,, \qquad L_{\pm1} = \frac{1}{2}\left(aH - \frac{K}{a} \mp i\tilde{D}\right)\,, \tag{3.6}$$

[8]The definition of dilatation in [22] is different from the one in [21]. The dilatation is D in [21] and $\tilde{D}$ in [22].

we can find the SL(2,$\mathbb{R}$) algebra in the Virasoro form,

$$[L_1, L_{-1}] = 2L_0\,, \qquad [L_0, L_{\pm 1}] = \mp L_{\pm 1}\,.$$

Here L_0 is the same as R, i.e., $L_0 = R$. Then the eigen-value of L_0 (R) gives the scaling dimension of the field and hence the energy eigen-value is nothing but the scaling dimension. The ground state in terms of R gives a primary field, and the excited states are secondary fields because L_{-1} gives the excited states. Eventually this fact matches the argument for the spectrum of a scalar field on AdS$_2$ [27, 23].[9]

Supersymmetric CQM. Finally we shall comment on the supersymmetric case. Including the fermionic degrees of freedom, the bosonic symmetry SL(2)×SO(3)×SO(5) is enhanced to the supergroup OSp($4^*|4$). The action of the supersymmetric CQM related to the supergroup is discussed in [29]. The action in [29] would be helpful for our purpose.

3.3 Wave functions from normalizable modes on AdS$_2$

Our argument can be further confirmed by computing a two-point function in the CQM from the degrees of freedom in the bulk AdS$_2$, i.e., by extracting the wave function from (2.8). The recipe has already been given in [23]. The variable in the dual CQM can be defined from (2.8) as follows:

$$\varphi^{\Delta}(t) \equiv \lim_{\rho\to\pi/2} \frac{\Gamma(2\Delta)}{c(\Delta)} (\cos\rho)^{-\Delta}\phi^{\Delta}(t,\rho)\,.$$

In terms of the modes, we can find that

$$\beta_n^{\Delta}(t) \equiv \lim_{\rho\to\pi/2} \frac{\Gamma(2\Delta)}{c(\Delta)} (\cos\rho)^{-\Delta}\phi_n^{\Delta}(t,\rho) = \sqrt{\frac{\Gamma(n+2\Delta)}{n!}}\,\mathrm{e}^{-i(n+\Delta)(t+\pi/2)}\,.$$

Then the two-point function can be computed from the boundary behavior of the Wightman function

$$\langle 0|\phi^{\Delta}(t_1,\rho_1)\phi^{\Delta}(t_2,\rho_2)|0\rangle$$

on the bulk AdS$_2$.

The two-point function $\langle\varphi^{\Delta}(t_1)\varphi^{\Delta}(t_2)\rangle$ is evaluated as follows:

$$\begin{aligned}\langle\varphi^{\Delta}(t_1)\varphi^{\Delta}(t_2)\rangle &= \lim_{\rho_1,\rho_2\to\pi/2} \left(\frac{\Gamma(2\Delta)}{c(\Delta)}\right)^2 (\cos\rho)^{-2\Delta}\langle\phi^{\Delta}(t_1,\rho_1)\phi^{\Delta}(t_2,\rho_2)\rangle \\ &= \sum_{n=0}^{\infty}\beta_n(t_1)\bar{\beta}_n(t_2) = \frac{\Gamma(2\Delta)\mathrm{e}^{i\Delta(t_1+t_2)}}{(\mathrm{e}^{it_1}-\mathrm{e}^{it_2})^{2\Delta}}\,. \end{aligned} \tag{3.7}$$

In the above computation the following formulae have been utilized:

$$\sum_{n=0}^{\infty}\frac{\Gamma(n+2\Delta)}{n!}x^n = \frac{\Gamma(2\Delta)}{(1-x)^{2\Delta}}\,, \qquad \Gamma(n+1) = n!\,.$$

[9]See also [28] for an earlier work on a two-dimensional black hole/CQM.

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The expression of (3.7) is written in Lorentzian signature and it needs some algebra to find a familiar form. Let us first perform the Wick rotation: $t = -it_{\rm E}$. Then EAdS$_2$ with Poincare metric,

$$ds^2 = \frac{d\tau_{\rm P}^2 + dz^2}{x^2}$$

is related to the Lorentzian metric (2.2) via the following coordinate transformation:

$$\tau_{\rm P} = {\rm e}^{t_{\rm E}} \sin\rho\,, \qquad z = {\rm e}^{t_{\rm E}} \cos\rho\,.$$

As approaching the boundary $z \to 0$, the Poincare time behaves as $\tau_{\rm P} \to {\rm e}^{t_{\rm E}}$. The conformal field $\varphi^{\Delta}(t)$ has the conformal weight Δ and transforms to $\hat{\varphi}(x)$ according to

$$\varphi^{\Delta}(t)(dt)^{\Delta} = \hat{\varphi}^{\Delta}(\tau_{\rm P})(d\tau_{\rm P})^{\Delta}\,.$$

Then we can obtain the correlation function

$$\langle \hat{\varphi}^{\Delta}(\tau_{P1})\hat{\varphi}^{\Delta}(\tau_{P2})\rangle = \frac{\Gamma(2\Delta)}{(\tau_{\rm P1} - \tau_{\rm P2})^{2\Delta}}\,.$$

This is nothing but the two-point function of the corresponding CQM [21].

From the above argument it is natural to look for the GKPW-type relation [2] as the next step. Then we have to consider NN modes, which are relevant to the operator insertions in the boundary theory. It will be discussed in detail in the next section.

4. Holography of NR string and Wilson line

In this section we shall discuss a GKPW-type relation [2] between the NR string and a Wilson line with local operator insertions, by considering NN modes of the NR string. We will work in Euclidean signature here.

In the first place we remember that the NR string action (2.1) can also be obtained as a semiclassical approximation around an AdS$_2$ solution corresponding to a Wilson loop [14, 16, 17]. Then we have to be careful for a field redefinition of the variables. This is sensitive to the order of the divergence of NN modes of (2.1) near at the boundary. Finally we propose a GKPW-type relation and clarify the related operator insertion in the gauge-theory side.

4.1 NR string from semiclassical limit

Let us remember the semiclassical approximation of the full AdS superstring around a static AdS$_2$ solution. Note that the AdS$_2$ classical solution satisfies the equations of motion obtained from the full action. Here we are confined to the bosonic part for simplicity and take the Nambu-Goto (NG) formulation with a static gauge.

The bosonic NG action is given by

$$S_{\rm NG} = \frac{\sqrt{\lambda}}{2\pi}\int d^2\sigma\,\sqrt{\det g}\,, \qquad g_{ij} = \partial_i X^M \partial_j X^N G_{MN}$$

in the Euclidean Poincare coordinates

$$ds^2 = G_{MN} dX^M dX^N = \frac{1}{z^2}\left(dX^m dX^m + dz^2\right) + d\Omega_5^2\,.$$

Here the AdS radius R_0 is already absorbed into the definition of 't Hooft coupling $\lambda \equiv N g_{\rm YM}^2 = R_0^4/\alpha'^2$.

Next the action can be expanded around the static classical solution as follows:

$$X^0 = \tau\,, \quad z = \sigma\,, \quad X^a = 0 + \sqrt{2\pi}\lambda^{-1/4}\tilde{x}^a\,, \quad Y^{a'} = 0 + \sqrt{2\pi}\lambda^{-1/4}\tilde{y}^{a'}\,,$$

where $Y^{a'}$ is the tangent coordinate on S^5. Then the induced metric can be expanded as

$$\begin{aligned} g_{ij} &= g_{0ij} + 2\pi\lambda^{-1/2}\left[\frac{1}{\sigma^2}\partial_i\tilde{x}^a\partial_j\tilde{x}^a + \partial_i\tilde{y}^{a'}\partial_j\tilde{y}^{a'}\right] + \cdots\,, \\ g_{0ij} &= \frac{1}{\sigma^2}\delta_{ij}\,. \end{aligned}$$

The resulting action is given by

$$\begin{aligned} S_{\rm NG} &= S_{(0)} + S_{(2)} + \mathcal{O}(\lambda^{-1/4})\,, \\ S_{(0)} &= \frac{\sqrt{\lambda}}{2\pi}\int d^2\sigma\,\sqrt{\det g_0} = \frac{\sqrt{\lambda}}{2\pi}\int d\tau\,\frac{1}{\epsilon}\,, \qquad (4.1) \\ S_{(2)} &= \frac{1}{2}\int d^2\sigma\,\sqrt{\det g_0}g_0^{ij}\left(\frac{1}{\sigma^2}\partial_i\tilde{x}^a\partial_j\tilde{x}^a + \partial_i\tilde{y}^{a'}\partial_j\tilde{y}^{a'}\right)\,. \qquad (4.2) \end{aligned}$$

Here $S_{(i)}$ $(i = 0, 1, 2, \ldots)$ denote the action with the i-th order quantum fluctuations. For the evaluation of $S_{(0)}$ the cut off ϵ has been introduced. With the equations of motion, the first-order contribution should vanish, i.e., $S_{(1)} = 0$, and hence it will not be touched below.

The zero-th order part gives a volume factor and it diverges. We can treat well this divergence by adding a boundary term. It will be discussed in the next subsection. As a result, for the classical solution concerned here, $S_{(0)}$ should vanish.

When the 't Hooft coupling λ is taken to be sufficiently large and the fluctuations are not divergent, the higher-order terms can be ignored. Then the approximation keeping the leading terms with lower order is valid. That is, the leading contribution is the quadratic fluctuations described by $S_{(2)}$.

By performing a field redefinition,

$$x^a = \frac{1}{\sigma}\tilde{x}^a\,, \quad y^{a'} = \tilde{y}^{a'}\,, \qquad (4.3)$$

the second order action can be rewritten as

$$S_{(2)} = S_{\rm NR} + \frac{1}{2}\int d^2\sigma\,\partial_\sigma\left(\frac{1}{\sigma}x^a x^a\right) \qquad (4.4)$$

where

$$S_{\rm NR} = \frac{1}{2}\int d^2\sigma\,\sqrt{\det g_0}\left[g_0^{ij}\left(\partial_i x^a\partial_j x^a + \partial_i y^{a'}\partial_j y^{a'}\right) + 2x^a x^a\right]\,. \qquad (4.5)$$

Thus the action $S_{(2)}$ is nothing but the bosonic action $S_{\rm NR}$ of the NR string discussed in the previous subsection, up to the last, surface term. The existence of this surface term will play an important role in our argument later in section 4.3.

The computation here will be available to estimate the magnitude of the fluctuations around the classical solution when we consider NN modes later.

4.2 Notes on boundary conditions

As is well known, there should be a divergence in the classical action for a classical sting solution corresponding to a Wilson loop [12]. Hence a regularization is necessary and then an appropriate boundary condition has to be imposed to remove the cut-off dependence [30].

Here it would possibly be interesting to see a different method to remove the divergence. This is to introduce a coupling to a constant NS-NS B-field into the string action [11]. This method has been utilized instead of the standard techniques [30]. Hereafter we will argue that the two methods may be equivalent.

Let us first remember the standard method given in [30]. It is easy to see that

$$\delta S_{\rm NG} = \int d\tau P^\sigma_M \delta X^M|_{\sigma=0}\,, \qquad P^\sigma_M = \frac{\partial \mathcal{L}}{\partial(\partial_\sigma X^M)}\,,$$

where the equations of motion have been used. It implies that the action $S_{\rm NG}$ is a functional of X^M at the boundary. On the other hand, the Wilson loop is a functional of X^m and $\dot{Y}^{m'}$, where $Y^{m'}$ are Cartesian coordinates on R^6:

$$(dY^{m'})^2 = dz^2 + z^2 d\Omega_5^2\,.$$

Hence it leads us to consider the Legendre transformation

$$S = S_{\rm NG} + S_L\,, \qquad S_L = -\int d\tau P^\sigma_{m'} Y^{m'}|_{\sigma=0}\,, \tag{4.6}$$

and then S is a functional of X^m and $\dot{Y}^{m'}$ at the boundary. The S_L is evaluated in the static gauge as

$$S_L = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{2}\frac{\partial_\sigma z^2}{z^2}\Big|_{\sigma=0} = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\epsilon}\,. \tag{4.7}$$

This term cancels out $S_{(0)}$ in (4.1) as expected.

Next we consider another method to introduce a constant B-field [11]. It gives $H = dB = 0$ and does not change the AdS$_5\times$S^5 background. Thus we may include a coupling to the B-field

$$S_B = \frac{1}{2\pi\alpha'}\int {}^*B = \frac{1}{2\pi\alpha'}\int d^2\sigma \epsilon^{ij}\partial_i X^M \partial_j X^N B_{MN}\,. \tag{4.8}$$

Let us consider the following closed two-form

$$B = -R_0^2 E^0 \wedge E^z = -\frac{R_0^2}{z^2} dX^0 \wedge dz\,, \tag{4.9}$$

where E is a vielbein.[10] Then (4.8) can be rewritten as

$$S_B = \frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\sigma}\bigg|_{\sigma=0}^{\sigma=\infty} = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\epsilon}\,. \tag{4.10}$$

This cancels out $S_{(0)}$ in (4.1).

Thus we can conclude that the two methods work well to cancel the divergence coming from $S_{(0)}$. In both cases we have treated the additional terms, S_L and S_B, as classical contributions. This is because these terms are added as background configurations which do not fluctuate. Therefore the fluctuations originate only from $S_{\rm NG}$.

Furthermore the Legendre transformation of $S_{\rm NG} + S_B$ may be considered by adding

$$S_L = -\int d\tau P_z^\sigma z|_{\sigma=0}\,, \qquad P_z^\sigma \equiv \frac{\partial(\mathcal{L}_{\rm NG} + \mathcal{L}_B)}{\partial(\partial_\sigma z)}\,.$$

Then it can be easily shown that $P_z^\sigma = 0$ and S_L should vanish. That is, the redundant divergent term does not arise.

In the next section 4.3, we evaluate the classical value of the quadratic action, $S_{(2)}[\Phi_0]$ by substituting the NN solution into the $S_{(2)}[\Phi]$. This should not be confused with $S_{(0)}$, which now corresponds to a Wilson loop. Finally, in section 4.4, we discuss a holographic interpretation of $S_{(2)}[\Phi_0]$.

4.3 NN modes on AdS_2

In the Euclidean case there exists NN solution only, while in the Lorentzian case normalizable solutions should be considered as well as NN modes. Hence the Lorentzian case is more complicated than the Euclidean case. The advantage in Euclidean signature was advocated in [2].

From now on we shall discuss NN solutions of the classical equation of motion derived from $S_{(2)}$. In Euclidean signature the world-sheet metric of EAdS_2 is given by

$$ds^2 = \frac{1}{\sigma^2}(d\tau^2 + d\sigma^2)\,.$$

This is a two-dimensional Poincare metric and the boundary is at $\sigma = 0$.

The NN solution $\Phi^I(\sigma,\tau)$ is specified by the behavior near the boundary

$$\Phi^I(\sigma,\tau) \to \sigma^{1-\Delta}\Phi_0^I(\tau) \qquad (\sigma \to 0)\,, \tag{4.11}$$

where the index I describes the eight transverse directions and $\Phi^I = (x^a, y^{a'})$. The Δ is fixed through the mass $m_{\rm B}$ of the variable Φ^I by

$$\Delta = \frac{1}{2}\left(1 + \sqrt{1 + 4R_0^2 m_{\rm B}^2}\right)\,,$$

[10] Here one may suspect here that a special B-field is taken by hand. But it would be fixed from the BPS condition for the classical solution. In fact, it could be done in the case of flat space [31]. It would be nice to consider the same analysis for the AdS case.

and it implicitly depends on the index I. From this behavior we find that x^a diverges near the boundary since $\Delta = 2$ for x^a. Then one might think that the divergence would break the semiclassical approximation around the AdS_2 solution.

But it is worth noting that the divergence of x^a does not imply that the fluctuation around the AdS_2 solution $\tilde{x}^a$ should also diverge. This is mainly because x^a and $\tilde{x}^a$ are related by (4.3): $x^a = \frac{1}{\sigma}\tilde{x}^a$. Actually, $\tilde{x}^a$ does not diverge as we will see below, and the semiclassical approximation may still be valid even in the presence of the NN modes.

The NN mode Φ^I is completely determined from the boundary value $\Phi_0^I(\tau)$ as follows:

$$\Phi^I(\sigma,\tau) = \int d\tau'\, K_\Delta(\sigma,\tau;\tau')\Phi_0^I(\tau'). \tag{4.12}$$

Here $K_\Delta(\sigma,\tau;\tau')$ is the bulk-to-boundary propagator [33, 32] defined as

$$K_\Delta(\sigma,\tau;\tau') \equiv \pi^{-1/2}\frac{\Gamma(\Delta)}{\Gamma\left(\Delta-\frac{1}{2}\right)}\frac{\sigma^\Delta}{(\sigma^2+(\tau-\tau')^2)^\Delta}. \tag{4.13}$$

Let us consider the surface term in $S_{(2)}$ given in (4.4). Its classical value is

$$-\frac{1}{2}\int d\tau \frac{1}{\epsilon^3}(x_0^a)^2\,, \tag{4.14}$$

where x_0^a is defined from the asymptotic behavior around the boundary

$$x^a \to \frac{1}{\epsilon}x_0^a \qquad (\sigma\to\epsilon)\,.$$

On the other hand, by integrating by part S_{NR} in (4.5) can be rewritten as

$$\begin{aligned} S_{\mathrm{NR}} = &-\frac{1}{2}\int d^2\sigma\,\left[x^a\partial^2x^a + y^{a'}\partial^2y^{a'} + \frac{2}{\sigma^2}x^ax^a\right] \\ &+\frac{1}{2}\int d^2\sigma\,\partial_\sigma\left(x^a\partial_\sigma x^a + y^{a'}\partial_\sigma y^{a'}\right) \end{aligned} \tag{4.15}$$

The contribution of the surface term is evaluated as

$$\frac{1}{2}\int d\tau\,\frac{1}{\epsilon^3}x_0^2\,,$$

and it cancels out (4.14). Thus $S_{(2)}$ is equivalent to the first line in (4.15). That is,

$$S_{(2)} = -\frac{1}{2}\int d^2\sigma\,\left[x^a\partial^2x^a + y^{a'}\partial^2y^{a'} + \frac{2}{\sigma^2}x^ax^a\right]. \tag{4.16}$$

The convergence of the σ-integration in (4.16) gives the precise unitarity bound on the dimension of a scalar [33]. Hence $S_{(2)}$ may be regarded as the precise action of the NR string including the boundary term to ensure the cancellation of the divergence at the boundary.

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In fact, $S_{(2)}$ in (4.2) is nothing but the proposed action to define the two-point function [33].[11] Then the classical value of $S_{(2)}$ with $\tilde{\Phi}^I \equiv (\tilde{x}^a, \tilde{y}^{a'})$ is given by

$$S_{(2)} = -\sum_{I=1}^{8} \lim_{\sigma\to 0} \sigma^{2-2\Delta} \frac{1}{2} \int d\tau\, \tilde{\Phi}^I \partial_\sigma \tilde{\Phi}^I \,. \tag{4.17}$$

By using (4.12), the resulting value of $S_{(2)}$ is evaluated as [33]

$$S_{(2)}[\Phi_0] = -\sum_{I=1}^{8} \left(\Delta - \frac{1}{2}\right) \pi^{-1/2} \frac{\Gamma(\Delta)}{\Gamma\left(\Delta - \frac{1}{2}\right)} \int d\tau \int d\tau' \, \frac{\Phi_0^I(\tau)\Phi_0^I(\tau')}{(\tau-\tau')^{2\Delta}} \,. \tag{4.18}$$

In the next we will discuss a holographic relation between (4.18) and a source term insertion on a straight Wilson line.

4.4 GKPW-type relation

Let us argue the role played by the quadratic action $S_{(2)}$ in the dual gauge-theory side from now on. For simplicity we restrict ourselves to the bosonic part again.

We begin our argument with the holographic relation *without* the quadratic fluctuations. That is, we focus upon $S_{(0)}$. For the straight Wilson line

$$W \equiv \mathrm{Tr} P \mathrm{e}^{\int dt\,(iA_0 + \phi_6)} \,,$$

the following relation is well known:

$$\langle W \rangle = \mathrm{e}^{-S_{(0)}} = 1 \,. \tag{4.19}$$

Note that $S_{(0)} = 0$ for the straight line.

The GKPW relation says that the right-hand side of (4.19) is the leading term of the semiclassical string partition function around the static AdS_2 solution corresponding to the straight Wilson line. Thus we see that

$$Z_{\rm string} = \int [d\Phi] \; \mathrm{e}^{-S_{\rm full}[\Phi]} \approx \mathrm{e}^{-S_{(0)}} = 1 \,. \tag{4.20}$$

Here $S_{(0)}$ has been evaluated by putting $\Phi = \Phi_{\rm cl}$ into the action $S_{\rm full}[\Phi]$, where $\Phi_{\rm cl}$ describes the classical solution corresponding to the 1/2 BPS Wilson line.

On the other hand, considering the semiclassical expansion around the classical solution in the string side, the Wilson loop corresponding to the string classical solution is inserted into the partition function of $\mathcal{N} = 4$ SYM,

$$Z_{\rm SYM} = \int [dA][d\phi]\, \mathrm{e}^{-S_{\rm SYM}[A,\phi]} \longrightarrow \int [dA][d\phi]\, W\, \mathrm{e}^{-S_{\rm SYM}[A,\phi]} = \langle W \rangle \,. \tag{4.21}$$

Thus (4.19) gives a piece of evidence for the conjectured relation

$$Z_{\rm SYM} = Z_{\rm string} \,,$$

[11] As noted there, for $\Delta \geq 3/2$, there still remain boundary divergences. They may correspond to contact terms in the Wilson loop expansion including δx in appendix D.

at the leading order of the semiclassical approximation.

Then the next problem to be considered is to add the quadratic action $S_{(2)}$ to (4.19). It is an easy task to add the contribution of $S_{(2)}$ to (4.20). The field Φ should be decomposed into the classical solution and the fluctuation like $\Phi = \Phi_{\rm cl} + \Phi_{\rm fl}$, and the full action $S_{\rm full}$ is expanded with respect to the fluctuations $\Phi_{\rm fl}$. The resulting expression is

$$Z_{\rm string} \approx {\rm e}^{-S_{(0)}} \int [d\Phi_{\rm fl}]\, {\rm e}^{-S_{(2)}[\Phi_{\rm fl}]} = \int [d\Phi_{\rm fl}]\, {\rm e}^{-S_{(2)}[\Phi_{\rm fl}]} \,. \tag{4.22}$$

Thus the string partition function is approximated by a path integral in terms of the fluctuation.

Now we shall discuss a further semiclassical approximation to (4.22) . The quadratic action $S_{(2)}$ describes a collection of free theories on AdS_2 . Then the GKPW relation may be applied to the world-sheet theory on the AdS_2 like in a scalar field theory on AdS_5 .

It would be helpful to remember some arguments for the Lorentzian AdS/CFT correspondence [19]. In the Lorentzian AdS/CFT, $\Phi_{\rm fl}$ has to be regarded as the sum of NN mode $\Phi_{\rm NN}$ and normalizable mode $\Phi_{\rm N}$ as follows: [19]

$$\Phi_{\rm fl} = \Phi_{\rm NN} + \Phi_{\rm N} \,.$$

Here $\Phi_{\rm NN}$ is characterized by the boundary value Φ_0 . Then the NN and normalizable modes should be treated as the background and fluctuation, respectively. Note that the result of [14] is reproduced by setting that $\Phi_{\rm NN} = 0$.

In our context this should be the so-called "second semiclassical approximation" or "double holography." It is not so surprising to see the holography of this type since other examples are already known in the case of defect CFT [24]. Observing that the AdS_2 solution is a kind of probe inserted in the bulk, such as probe D-branes in the bulk AdS_5 , it is quite natural to see the holography even for the present case.

There however exists no normalizable solution (i.e., $\Phi_{\rm N} = 0$) in Euclidean signature. Thus we propose the following holographic relation

$$\left\langle {\rm Tr} P\left[{\rm e}^{\int dt\,(iA_0+\phi_6)} \cdot {\rm e}^{\int dt\, \mathcal{O}_I\cdot\Phi_0^I}\right]\right\rangle = {\rm e}^{-S_{(2)}[\Phi_0]} \,. \tag{4.23}$$

The $S_{(2)}[\Phi_0]$ is obtained by putting the NN mode $\Phi_{\rm fl} = \Phi_{\rm NN}$ into the $S_{(2)}[\Phi_{\rm fl}]$ and it is given by (4.18). Note that the boundary values couple to the operators in one dimension rather than four dimensions. The other three coordinates are fixed as the same as the position of the Wilson line.

By taking the functional derivatives in terms of the sources Φ_0's and then setting that $\Phi_0 = 0$, it is possible to produce correlation functions of the operators $\mathcal{O}$'s inserted on the one-dimensional Wilson line. The gravity side is a collection of the NN modes of the scalar fields on the AdS_2 and so it is natural to argue that the correlation functions should be related to the CQM via $\mathrm{AdS}_2/\mathrm{CFT}_1$.

4.5 The coupled operators

So far we have not specified what operators should be inserted as $\mathcal{O}_I$. The coupled operators have already been clarified in our previous works [16, 17].

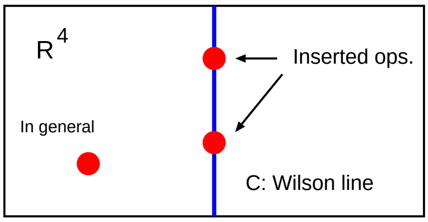

Figure 2: The restriction of the insertion points of the operators onto the Wilson line C. In general the operators may be inserted everywhere on $\mathbb{R}^4$.

The coupled operators $\mathcal{O}$ are

$$\mathcal{O}_I = \begin{cases} iF_{a0} + D_a\phi_6 & (a = 1,2,3) \\ \phi^{a'} & (a' = 1,\ldots 5) \end{cases} .$$

The operators with $\Delta = 2$ and $\Delta = 1$ correspond to massive and massless scalars, respectively. This is well supported from the GKPW relation between mass of scalar fields and conformal dimensions for AdS_2 [2],

$$\Delta = \frac{1}{2}\left(1 + \sqrt{1 + 4R_0^2 m_{\mathrm{B}}^2}\right) . \tag{4.24}$$

With this relation (4.24) we obtain that

$$\Delta = 2 \quad \text{for} \quad m_{\mathrm{B}}^2 = 2/R_0^2 , \qquad \Delta = 1 \quad \text{for} \quad m_{\mathrm{B}}^2 = 0 .$$

The GKPW relation has been used for the AdS_2 case above. One can figure out this as a restriction of the positions of the operators inserted at the boundary. Taking AdS_2 rather than AdS_5, the operators are inserted just on the Wilson line rather than everywhere in $\mathbb{R}^4$. For an intuitive picture see figure 2. So far the bosonic insertion has been considered but the fermionic insertion also can be discussed. The fermionic insertion is discussed in [17] and it is Ψ satisfying $P_+\Psi = \Psi$. Thus it has the eight physical components and its conformal dimension is $3/2$.

From another viewpoint, the source term insertion may be regarded as a deformation of the straight Wilson line, which was discussed in [16] by following [34]. In our previous works [16, 17] the physical meaning of the small fluctuation δC around the straight Wilson line C_0 was unclear, but now one can realize that δC should be given by adding the source terms. In fact, the Wilson loop expansion discussed in [16] can be reproduced by expanding the source term up to the contact terms (For the second order expansion of the Wilson line see appendix D). For the first order it can easily be checked by identifying Φ_0 with the deformations δx^a and $\delta \dot{y}^{a'}$.

In the string-theory side 16 linear supersymmetries are possessed by the quadratic action $S_{(2)}$, so the same amount of supersymmetries should be preserved even after the source term has been inserted. This is shown in appendix E.

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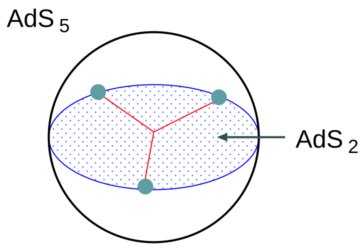

Figure 3: A contribution of the third-order fluctuation.

4.6 A comment on higher-order fluctuations

Finally let us comment on higher-order fluctuations, e.g., $S_{(3)}$.

In the Euclidean case the fluctuations around the classical solution corresponding to the Wilson line are nothing but the NN modes. Then higher order fluctuations depend on the boundary value Φ_0 like $S_{(i)}[\Phi_0]$. For example, a contribution of the third-order fluctuation $S_{(3)}[\Phi_0]$ gives a triple coupling on the AdS_2 world-sheet as depicted in figure 3.

The higher-order fluctuations are not discussed in this paper, but it would be interesting to consider them carefully as a future direction.

5. Conclusion and discussion

We have discussed a holographic dual of the NR string on $\mathrm{AdS}_5\times\mathrm{S}^5$. The physical spectrum of the NR string [18] completely agrees with that of a one-dimensional CQM. Then the wave functions and the two-point function of the CQM can be reproduced from the physical modes of the NR string. From this result we have argued that an $\mathrm{AdS}_2/\mathrm{CFT}_1$ may be realized in $\mathrm{AdS}_5/\mathrm{CFT}_4$. In Euclidean signature there exist only NN solutions for the equation of motion of the NR string. Then the boundary values related to a source term giving the operator insertions on the Wilson line. The source term may be regarded as a deformation of the Wilson loop. Thus we have proposed a GKPW-type relation between the NR string and the deformed Wilson line by the source term insertion.

There remains an open problem that is to derive the CQM action directly from $\mathcal{N}=4$ SYM or the D-brane setup. The NR limit is closely related to the Higgs mechanism in $\mathcal{N}=4$ SYM. This would be obvious by considering the D3-brane setup before taking the near-horizon limit. The fluctuations around the AdS_2 solution correspond to those around an infinitely long open string. Thus the CQM may describe a quantum mechanics of the probe quarks supplied from the long string. It would be interesting to study the origin of the CQM in this direction.

It would also be nice to consider a further generalization of our arguments. The first example is to discuss a circular case. The quadratic action has already been discussed in [14] and the corresponding Wilson loop expansion has been done in [17]. The remaining problem is to derive normalizable modes on AdS_2 by using the Poincare disk, instead of the strip considered in [18]. It may be expected that the dual CQM is defined on the circle given by the Wilson loop.

The second example is to consider higher-dimensional representation instead of the fundamental representation. Then the Wilson loop should be replaced by giant Wilson loops [35] and we have to use D-brane actions on $AdS_5 \times S^5$ instead of the string action. From the viewpoint of the gauge theory the operator insertion should not be modified. The only difference is the representation of the trace. Thus the quadratic fluctuations around the giant Wilson loops should behave as those around AdS_2 in the vicinity of the boundary. Thus we can easily guess the agreement. We will report on the detail computation in the near future [36].

We hope that our research may be able to shed light on AdS_2/CFT_1 correspondence.

Acknowledgments

The authors would like to thank N. Iizuka, A. Jevicki, Y. Kazama, A. Miwa, Y. Okawa, T. Okuda, T. Takayanagi, D. Trancanelli, A. Tsuji, N. Yokoi and T. Yoneya. They also thank the YITP workshop YITP-W-07-05 on "String Theory and Quantum Field Theory" and the international workshop "Progress of String Theory and Quantum Field Theory" (2007/12/7-10, Osaka City University). Discussions there were useful to complete this work.

The work of M.S. was supported in part by the Grant-in-Aid for Scientific Research (19540324, 19540304, 19540098) from the Ministry of Education, Science and Culture, Japan. The work of K.Y. was supported in part by JSPS Postdoctoral Fellowships for Research Abroad and the National Science Foundation under Grant No. NSF PHY05-51164.

A. NR limit in another coordinate system of $AdS_5 \times S^5$

The NR limit given in [11] is written with a complicated, unfamiliar metric. Here we shall rewrite the limit in terms of more familiar coordinate system.

The following coordinates for $AdS_5 \times S^5$, which keeps an AdS_2 factor explicitly, would be useful:

$$\begin{aligned} ds^2 =\ & R_0^2\left[\cosh^2 u\,(-\cosh^2 x\, dt^2 + dx^2) + du^2 + \sinh^2 u\,(d\varphi_1^2 + \sin^2\phi_1 d\phi_2^2)\right] \\ & + R_0^2\left[d\gamma^2 + \cos^2\gamma\, d\varphi_3^2 + \sin^2\gamma\left(d\psi^2 + \cos^2\psi\, d\varphi_1^2 + \sin^2\psi\, d\varphi_2^2\right)\right]. \end{aligned} \tag{A.1}$$

Here the S^5 part is also represented by

$$\begin{aligned} X_1 + iX_2 &= \sin\gamma\cos\psi e^{i\varphi_1}, \qquad & X_3 + iX_4 &= \sin\gamma\sin\psi e^{i\varphi_2}, \\ X_5 + iX_6 &= \cos\gamma e^{i\varphi_3}, \qquad & X_1^2 + \cdots + X_6^2 &= 1. \end{aligned} \tag{A.2}$$

Then the NR limit in [11] can be taken as follows.

$$u = \frac{\tilde{u}}{R_0}, \qquad \gamma = \frac{r}{R_0}, \qquad \varphi_3 = \frac{\pi}{2} + \frac{y}{R_0}, \qquad R_0 \to \infty. \tag{A.3}$$

This limit keeps the AdS_2 factor in AdS_5 while the geometry around a point is closed up. Thus it respects an $SO(3)\times SO(5)$ symmetry preserved by the corresponding Wilson line.

The resulting metric is given by

$$\begin{aligned} ds^2 &= (R_0^2+\tilde{u}^2)(-\cosh^2 x\, dt^2+dx^2)+d\tilde{u}^2+\tilde{u}^2\,(d\phi_1^2+\sin^2\phi_1 d\phi_2^2) \\ &\quad +dr^2+dy^2+r^2\left(d\psi^2+\cos^2\psi\, d\varphi_1^2+\sin^2\psi\, d\varphi_2^2\right)\,, \end{aligned} \tag{A.4}$$

which is equipped with the divergent NS-NS two-form flux

$$B_{tx} = R_0^2 \cosh x\,. \tag{A.5}$$

It basically describes the geometry of $\mathrm{AdS}_2\times\mathbb{R}^8$, though the leading term is divergent. This is nothing but the metric obtained after taking the NR limit in [11].

B. A heuristic interpretation of the NR string action

The original derivation of the NR string is complicated and it seems difficult to figure out the essential point. So we will not repeat it but give a heuristic interpretation of the NR string action. In particular we aim at understanding how the mass terms can appear in the quadratic action. It would be important to have a rough image for the mechanism to generate the mass terms.

We start from the background (A.4) with (A.5) , which was obtained from $\mathrm{AdS}_5\times\mathrm{S}^5$ background with a constant NS-NS two-form flux after taking the NR limit (A.3) . For simplicity we restrict ourselves to the bosonic part.

The Nambu-Goto action on this background is given by

$$S^{(\mathrm{NR})} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,\sqrt{-\det g}+\frac{1}{2\pi\alpha'}\int d^2\sigma\, B_{tx}\,,$$

where the induced metric is given by

$$\begin{aligned} g_{ij} &= (R_0^2+(x^a)^2))g_{0ij}+\partial_i x^a\partial_j x^a+\partial_i y^{a'}\partial_j y^{a'}\,. \\ g_{0ij} &= \mathrm{diag}(-\cosh^2\sigma,1)\,, \end{aligned}$$

by using the static gauge

$$t=\tau\,,\qquad x=\sigma\,.$$

Here the coordinates have been rewritten as

$$\begin{aligned} d\tilde{u}^2+\tilde{u}^2\,(d\phi_1^2+\sin^2\phi_1 d\phi_2^2) &= \sum_{a=1}^{3} dx^a dx^a\,, \\ dr^2+dy^2+r^2\left(d\psi^2+\cos^2\psi\, d\varphi_1^2+\sin^2\psi\, d\varphi_2^2\right) &= \sum_{a'=1}^{5} dy^{a'}dy^{a'}\,. \end{aligned}$$

Then the action can be expanded as

$$S^{(\mathrm{NR})} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\det g_0}\, g_0^{ij}\left(\partial_i x^a\partial_j x^a+g_{0ij}(x^a)^2+\partial_i y^{a'}\partial_j y^{a'}\right)+\mathcal{O}\left(R_0^{-2}\right)\,.$$

The divergence in $R_0 \to \infty$ limit has been canceled out due to the presence of the NS-NS two-form (A.5). The higher-order terms also disappear in this limit. It is important to observe that the value of the mass 2 counts the number of the world-sheet coordinates, namely τ and σ (or the AdS_2 factor in the metric (A.4)). It would be easy to extend the above argument to NR D-brane cases [15, 16].

It is also helpful to remember the pp-wave case, where the mass term comes from the $(++)$-component of the metric, $ds^2 = -2dx^+dx^- + G_{++}(dx^+)^2 + \cdots$. In analogy to the pp-wave, $(dx^+)^2$ correspond to the AdS_2 factor in the present case.

Thus we have understood how the mass terms should come up in the action and the meaning of the value of the mass.

C. Some properties of Gegenbauer polynomial

Here we shall summarize some useful properties of Gegenbauer polynomial in investigating normalizable modes of a scalar field on AdS_2.

The Gegenbauer polynomial is represented by the hyper geometric function or Jacobi polynomial $P_\alpha^{(a,b)}(z)$ with $a = b = \lambda - 1/2$

$$\begin{aligned} C_\alpha^\lambda(z) &= \frac{\Gamma(\alpha+2\lambda)}{\Gamma(\alpha+1)\Gamma(2\lambda)} F\left(\alpha+2\lambda, -\alpha, \lambda+\frac{1}{2}; \frac{1-z}{2}\right) \\ &= \frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(2\lambda)} \frac{\Gamma(\alpha+2\lambda)}{\Gamma(\alpha+\lambda+\frac{1}{2})} P_\alpha^{(\lambda-1/2,\lambda-1/2)}(z)\,. \end{aligned}$$

It satisfies the following hyper-geometric differential equation

$$\left[(1-z^2)\frac{d^2}{dz^2} - (2\lambda+1)\frac{d}{dz} + \alpha(\alpha+2\lambda)\right] C_\alpha^\lambda = 0\,.$$

When λ is real and $\lambda > -1/2$, the Gegenbauer polynomials satisfy the orthonormal condition given by

$$\int_{-1}^{1} dx\,(1-x^2)^{\lambda-1/2} C_m^\lambda(x) C_n^\lambda(x) = \frac{\pi\Gamma(n+2\lambda)}{2^{2\lambda-1} n!(\lambda+n)\Gamma(\lambda)^2}\delta_{m,n}\,.$$

The Gegenbauer polynomials are also given by the generating function

$$\frac{1}{(1-2zt+t^2)^\lambda} = \sum_{\alpha=0}^{\infty} C_\alpha^\lambda(z) t^\alpha\,.$$

The first few Gegenbauer polynomials are

$$\begin{aligned} &C_0^\lambda(z) = 1\,, \qquad C_1^\lambda(z) = 2\lambda z\,, \qquad C_2^\lambda(z) = -\lambda + 2\lambda(1+\lambda)z^2\,, \\ &C_3^\lambda(z) = -2\lambda(1+\lambda)z + \frac{4}{3}\lambda(1+\lambda)(2+\lambda)z^3\,. \end{aligned}$$

D. Wilson loop expansion at the second order

Let us discuss a Wilson loop expansion at the second order with respect to a small deformation of the loop.

A Wilson loop with a contour C is described by

$$W(C) = \mathrm{Tr}\mathcal{W}_{u_1}^{u_2}\,, \quad \mathcal{W}_{u_1}^{u_2} = P\exp\int_{u_1}^{u_2} ds\,\left(iA_\mu(x(s))\dot{x}^\mu(s) + \phi_i(x(s))\dot{y}^i\right)\,.$$

The locally supersymmetry condition

$$(\dot{x}^\mu)^2 - (\dot{y}^i)^2 = 0 \tag{D.1}$$

can be viewed as the integrability condition of the super-invariance of $W(C)$:

$$(i\Gamma_\mu\dot{x}^\mu + \Gamma_i\dot{y}^i)\epsilon = 0\,.$$

Let us consider a small deformation of $W(C)$ by taking C as $C = C_0 + \delta C$:

$$x^\mu = x^\mu_{C_0} + \delta x^\mu\,, \qquad y^i = y^i_{C_0} + \delta y^i\,.$$

Then $W(C)$ can be expanded as

$$\begin{aligned} W(C) &= W(C_0) + \int_{u_1}^{u_2} ds\left[\delta x^\mu(s)\frac{\delta W(C)}{\delta x^\mu(s)}\Big|_{C_0} + \delta\dot{y}^i(s)\frac{\delta W(C)}{\delta\dot{y}^i(s)}\Big|_{C_0}\right] \\ &\quad + \int_{u_1}^{u_2} ds_1\int_{u_1}^{u_2} ds_2\left[\delta x^\mu(s_1)\delta x^\nu(s_2)\frac{\delta^2 W(C)}{\delta x^\mu(s_1)\delta x^\nu(s_2)}\Big|_{C_0} + \cdots\right] + \cdots\,. \end{aligned}$$

It is straightforward to derive[12]

$$\begin{aligned} \frac{\delta\mathcal{W}_{u_1}^{u_2}}{\delta x^\mu(s)} &= \mathcal{W}_{u_1}^{s} O_\mu(s)\mathcal{W}_s^{u_2} + \mathcal{W}_{u_1}^{u_2}(iA_\mu)_s\delta(u_2 - s) - (iA_\mu)_s\delta(u_1 - s)\mathcal{W}_{u_1}^{u_2}\,, \\ \frac{\delta\mathcal{W}_{u_1}^{u_2}}{\delta\dot{y}^i(s)} &= \mathcal{W}_{u_1}^{s} O_i(s)\mathcal{W}_s^{u_2}\,. \end{aligned} \tag{D.2}$$

Here we have introduced O_μ and O_i defined as, respectively,

$$O_\mu \equiv iF_{\mu\nu}\dot{x}^\nu + D_\mu\phi_i\dot{y}^i\,, \qquad O_i \equiv \phi_i\,,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ and $D_\mu\phi_i = \partial_\mu\phi_i + i[A_\mu, \phi_i]$.

In the first equation in (D.2), we have performed a partial-integration and used the following properties:

$$\partial_u\mathcal{W}_{u_1}^{u} = \mathcal{W}_{u_1}^{u}(iA_\mu\dot{x}^\mu + \phi_i\dot{y}^i)_u\,, \quad \partial_u\mathcal{W}_{u}^{u_2} = -(iA_\mu\dot{x}^\mu + \phi_i\dot{y}^i)_u\mathcal{W}_u^{u_2}\,. \tag{D.3}$$

From (D.2) one can read off the operator insertion preserving half of the supersymmetries. This is analogy with the pp-wave case [34].

[12]We denote $iA_\mu(x(s))$ as $(iA_\mu)_s$ for short.

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A 1/2 BPS straight Wilson line is realized by choosing C_0 as

$$C_0 : x^\mu = (s, 0, 0, 0)\,, \qquad \dot{y}^i = (0, 0, 0, 0, 0, 1)\,,$$

which satisfies the locally supersymmetry condition (D.1). Then $W(C_0)$ represents a straight line with $u_1 = -\infty$ and $u_2 = +\infty$, which is represented by

$$W(C_0) = \mathrm{Tr}\mathcal{W}_0{}^{u_2}_{u_1}\,, \qquad \mathcal{W}_0{}^{u_2}_{u_1} = \mathcal{W}^{u_2}_{u_1}|_{C_0} = P\exp\int_{u_1}^{u_2} du(iA_0 + \phi_6)_u\,.$$

From (D.2), the first order deformation of $W(C_0)$ is given by

$$\begin{aligned}
\frac{\delta W(C)}{\delta x^\mu(s)}\Big|_{C_0} &= \mathrm{Tr}\ \mathcal{W}_0{}^{s}_{u_1}\mathcal{O}_\mu(s)\mathcal{W}_0{}^{u_2}_{s}\,,\\
\frac{\delta W(C)}{\delta \dot{y}^i(s)}\Big|_{C_0} &= \mathrm{Tr}\ \mathcal{W}_0{}^{s}_{u_1}\mathcal{O}_i(s)\mathcal{W}_0{}^{u_2}_{s}\,,
\end{aligned}$$

where

$$\mathcal{O}_\mu = O_\mu|_{C_0} = iF_{\mu 0} + D_\mu\phi_6\,, \qquad \mathcal{O}_i = O_i|_{C_0} = \phi_i\,.$$

Then the second-order derivatives are

$$\begin{aligned}
\frac{\delta^2 W(C)}{\delta x^\mu(s_1)\delta x^\nu(s_2)}\Big|_{C_0} &= \mathrm{Tr}\Big[\mathcal{W}_0{}^{s_1}_{u_1}\mathcal{O}_\mu(s_1)\mathcal{W}_0{}^{s_2}_{s_1}\mathcal{O}_\nu(s_2)\mathcal{W}_0{}^{u_2}_{s_2}\\
&\quad + \mathcal{W}_0{}^{s_1}_{u_1}\big(iD_{(\mu}F_{\nu)0} + D_{(\mu}D_{\nu)}\phi_6\big)\mathcal{W}_0{}^{u_2}_{s_1}\delta(s_2 - s_1)\\
&\quad - \frac{1}{2}\mathcal{W}_0{}^{s_2}_{u_1} iF_{\mu\nu}(s_2)\partial_{s_2}\delta(s_2 - s_1)\mathcal{W}_0{}^{u_2}_{s_2}\\
&\quad + \frac{1}{2}\mathcal{W}_0{}^{s_1}_{u_1} iF_{\mu\nu}(s_1)\partial_{s_1}\delta(s_2 - s_1)\mathcal{W}_0{}^{u_2}_{s_1}\Big]\,,\\
\frac{\delta^2 W(C)}{\delta \dot{y}^i(s_1)\delta x^\mu(s_2)}\Big|_{C_0} &= \mathrm{Tr}\Big[\mathcal{W}_0{}^{s_1}_{u_1}\mathcal{O}_i(s_1)\mathcal{W}_0{}^{s_2}_{s_1}\mathcal{O}_\mu(s_2)\mathcal{W}_0{}^{u_2}_{s_2}\\
&\quad + \mathcal{W}_0{}^{s_2}_{u_1}(D_\mu\phi_i)_{s_2}\delta(s_2 - s_1)\mathcal{W}_0{}^{u_2}_{s_2}\Big]\,,\\
\frac{\delta^2 W(C)}{\delta x^\mu(s_1)\delta \dot{y}^i(s_2)}\Big|_{C_0} &= \mathrm{Tr}\Big[\mathcal{W}_0{}^{s_1}_{u_1}\mathcal{O}_\mu(s_1)\mathcal{W}_0{}^{s_2}_{s_1}\mathcal{O}_i(s_2)\mathcal{W}_0{}^{u_2}_{s_2}\\
&\quad + \mathcal{W}_0{}^{s_2}_{u_1}(D_\mu\phi_i)_{s_2}\delta(s_2 - s_1)\mathcal{W}_0{}^{u_2}_{s_2}\Big]\,,\\
\frac{\delta^2 W(C)}{\delta \dot{y}^i(s_1)\delta \dot{y}^j(s_2)}\Big|_{C_0} &= \mathrm{Tr}\Big[\mathcal{W}_0{}^{s_1}_{u_1}\mathcal{O}_i(s_1)\mathcal{W}_0{}^{s_2}_{s_1}\mathcal{O}_j(s_2)\mathcal{W}_0{}^{u_2}_{s_2}\Big]\,,
\end{aligned}$$

where $s_1 \leq s_2$ is assumed. We have used (D.2) and (D.3), and performed partial integrations. It is straightforward to derive higher order deformations, but we will not touch on them here.

Note that the above second-order deformations contain the contact terms which look like Schwinger terms. Now we are not sure for the physical interpretation of these terms in our context. The Wilson loop expansion coincides with the Wilson loop with the source term

$$\mathrm{Tr}P\left[\mathrm{e}^{\int dt(iA_0+\phi_6)}\,\mathrm{e}^{\int dt\mathcal{O}_I\Phi^I}\right]\,, \tag{D.4}$$

up to these contact terms.

E. Supersymmetry of deformed Wilson loop

We expect 16 supersymmetries preserved by the deformed Wilson line $W(C)$ from the result in the string-theory side. The purpose here is to show that it is really supersymmetric.

First of all, let us recall the relation between world-sheet fluctuations and NN modes. As seen in section 4.3, the world-sheet fluctuations

$$(\sqrt{2\pi}\lambda^{-1/4}\tilde{x}^a, \sqrt{2\pi}\lambda^{-1/4}\tilde{y}^{a'})$$

are small comparing to the classical contribution as $\lambda \to \infty$.

The NN modes are characterized by the behavior in the vicinity of the boundary as (4.11). As (x, y) and $(\tilde{x}, \tilde{y})$ are related by (4.3), we find

$$\tilde{x} \to x_0(\tau)\,, \qquad \tilde{y} \to y_0(\tau)\,,$$

as approaching the boundary. This implies that $(\tilde{x}, \tilde{y})$ do not grow large near the boundary. Thus the world-sheet fluctuations near the boundary

$$(\sqrt{2\pi}\lambda^{-1/4}x_0^a, \sqrt{2\pi}\lambda^{-1/4}y_0^{a'})$$

still remain small. Those may cause small deformations of the Wilson line.

Now we shall consider supersymmetries preserved by $W(C)$. The supersymmetry condition is given by the locally supersymmetry condition (D.1) imposed on the coordinates at a point on C. For the first-order deformation, the condition reads [16]

$$(\dot{x}^\mu_{C_0} + \delta\dot{x}^\mu)^2 - (\dot{y}^\mu_{C_0} + \delta\dot{y}^\mu)^2 = 2(\delta\dot{x}^0 - \delta\dot{y}^6) = 0\,.$$

Since $\delta\dot{x}^0 + \delta\dot{y}^6 = 0$ can be obtained by using SO(2) symmetry, we may choose

$$\delta x^0 = \delta\dot{y}^6 = 0\,. \tag{E.1}$$

Next it is the turn to consider the second-order deformation at a point. It corresponds to the terms with a δ-function in the Wilson line expansion and this should not be confused with two first-order deformations at two different points on C. Let us denote the second-order deformation as $(\delta^2 x^\mu, \delta^2 y^i)$. Then it is obvious that the condition is trivially satisfied as follows:

$$(\dot{x}^\mu_{C_0} + \delta\dot{x}^\mu + \delta^2\dot{x}^\mu)^2 - (\dot{y}^\mu_{C_0} + \delta\dot{y}^\mu + \delta^2\dot{y}^\mu)^2 = 2(\delta\dot{x}^0 - \delta\dot{y}^6) = 0\,,$$

because the second-order deformation is small comparing to the first-order deformation.

This is the case for higher-order deformations at a point and $W(C)$ coincides with (D.4) up to contact terms. Thus we have shown that $W(C)$ is supersymmetric under the condition (E.1). Inversely speaking, the consistency with the supersymmetries requires the condition (E.1).

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